

**BLUE PRINT FOR QUESTION PAPER PATTERN**  
**COURSE-III, ABSTRACT ALGEBRA**

<b>Unit</b>	<b>TOPIC</b>	<b>S.A.Q(including choice)</b>	<b>E.Q(including choice)</b>	<b>Total Marks</b>
I	Groups	2	2	30
II	Subgroups, Cosets & Lagrange's theorem	1	2	25
III	Normal Subgroups and Homomorphism	1	2	25
IV	Permutations and Cyclic groups	2	2	30
V	Rings	2	2	30
Total		8	10	140

**S.A.Q.** = Short answer questions (5 marks)

**E.Q.** = Essay questions (10 marks)

Short answer questions : 5 X 5 M = 25 M

Essay questions : 5 X 10 M = 50 M

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 Total Marks = 75 M  
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**CBCS/ SEMESTER SYSTEM**  
**(w.e.f. 2020-21 Admitted Batch)**  
**B.A./B.Sc. MATHEMATICS**  
**COURSE-III, ABSTRACT ALGEBRA**

**Time: 3Hrs**

**Max.Marks:75M**

**SECTION - A**

**Answer any FIVE questions. Each question carries FIVE marks 5 X 5 M=25 M**

1. Show that the set  $G = \{x/x - 2^a 3^b \text{ and } a, b \in \mathbb{Z}\}$  is a group under multiplication
2. Define order of an element. In a group G, prove that if  $a \in G$  then  $O(a) = O(a)^{-1}$ .
3. If H and K are two subgroups of a group G, then prove that  $HK$  is a subgroup  $\Leftrightarrow HK=KH$
4. If G is a group and H is a subgroup of index 2 in G then prove that H is a normal subgroup.
5. Examine whether the following permutations are even or odd

i)  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 1 & 4 & 3 & 2 & 5 & 7 & 8 & 9 \end{pmatrix}$

ii)  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 7 & 4 & 5 & 6 & 7 & 1 \end{pmatrix}$

6. Prove that a group of prime order is cyclic.
7. Prove that the characteristic of an integral domain is either prime or zero.
8. If F is a field then prove that  $\{0\}$  and F are the only ideals of F.

**SECTION - B**

**Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M**

9 a) Show that the set of  $n^{\text{th}}$  roots of unity forms an abelian group under multiplication.

(Or)

9 b) In a group G, for  $a, b \in G$ ,  $O(a)=5$ ,  $b \neq e$  and  $aba^{-1} = b^2$ . Find  $O(b)$ .

10 a) The Union of two subgroups is also a subgroup  $\Leftrightarrow$  one is contained in the other.

(Or)

b) State and prove Lagrange's theorem.

11 a) Prove that a subgroup  $H$  of a group  $G$  is a normal subgroup of  $G$  iff the product of two right cosets of  $H$  in  $G$  is again a right coset of  $H$  in  $G$ .

(Or)

11 b) State and prove fundamental theorem of homomorphisms of groups.

12 a) Let  $S_n$  be the symmetric group on  $n$  symbols and let  $A_n$  be the group of even permutations. Then show that  $A_n$  is normal in  $S_n$  and  $O(A_n) = \frac{1}{2}(n!)$

(Or)

12 b) Prove that every subgroup of a cyclic group is cyclic.

13 a) Prove that every finite integral domain is a field.

(Or)

13 b) Define principal ideal. Prove that every ideal of  $\mathbb{Z}$  is a principal ideal.